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ENG/20M

CSCE 531 Homework 3 and Homework 4

1. Determine whether is one-to-one if

This function is one-to-one. Every element in the domain maps to some unique in the range.

This is not one-to-one. For example, given , .

1. Determine whether is onto if

This function is not onto. For example,

This function is onto. We can map to if , we can map to positive numbers if , and we can map to negative numbers if .

1. Use the identities and , where is a sequence of real numbers, to compute .

First, .

In continuing, we see that .

Thus, .

1. Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit (show) a one-to-one correspondence between and that set.
   1. The set of integers not divisible by .

This is countably infinite. Here’s an example mapping:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | … |
| 0 | 1 | -1 | 2 | -2 | 4 | -4 | 5 | -5 | … |

* 1. The set of integers divisible by but not by .

This set is countably infinite. Here’s an example mapping:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 2 | 3 | … | 11 | 12 | 13 | 14 | 15 | … |
| 5 | -5 | 10 | … | 30 | -30 | 40 | -40 | 45 | … |

* 1. The set of real numbers with decimal representations of all s or s.

This set is uncountably infinite. We can always add a or a to our most precise real number to find a new real number.

1. For each of the following properties, give an example of two uncountable sets and such that is a set having that property.
   1. Finite
   2. Countably infinite
   3. Uncountable
2. Find all solutions, if any, to the following system of congruences.
3. Use Fermat’s little theorem and the Chinese remainder theorem to find
4. Use strong induction to prove that postage of cents can be formed using just -cent stamps and -cent stamps.

Basis step:

Inductive hypothesis:

Assume that, for all such that , holds.

Inductive step:

We know , and, if , then . Since , by the inductive hypothesis we know that is true. Because we can always add one -cent stamp, we can reach cents; in other words, we can always form a postage of at least cents.

By the principle of strong induction, we have proved the theorem.

1. Use strong induction to show that every positive integer can be written as a sum of distinct powers of two. *Hint: For the inductive step, consider even and odd numbers separately.*

Basis step:

Inductive hypothesis:

Assume that, for all such that , holds.

Inductive step:

We know that the theorem holds for all values less than or equal to . We also know that is either odd or even.

is odd:

In this case, is necessarily even. That means that all powers of two that sum to are also even. That is, is even. Because they’re all even, is not included in the list of terms that sum to . This implies that we can add to to reach ; that is,

.

is even:

If is even, we know that is an even, natural number. We also know that, under the inductive hypothesis, holds. Because , we can multiply both sides by to show that

,

which still consists of distinct powers of .

We’ve now shown that we can reach in all cases (that is, those where is even and those where is odd). By the principle of strong induction, we have proved the theorem.

1. Suppose that is a propositional function. Determine for which the statement must be true if
   1. is true and .

The set of all nonnegative integers divisible by

* 1. is true and .

The set of all nonnegative integers divisible by

* 1. and are true and .

The set of all nonnegative integers

* 1. is true and .

The set of all nonnegative integers except

1. Prove that for all where is the th Fibonacci number.

Base case:

Inductive hypothesis:

Assume holds.

Inductive step:

By the inductive hypothesis,

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By the definition of Fibonacci numbers,

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By the principle of mathematical induction, the theorem is proved.